Aharonov-Bohm effect and nucleon-nucleon phase shifts on the lattice

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One of the central goals of nuclear physics is to relate the successful phenomenological models developed throughout the years with the underlying fundamental theory of the strong interactions, QCD. Effective field theories are an important step in this direction, but they are inherently limited by the existence of low energy constants whose values are not determined by symmetries and have to be fit to experiment. The need is then obvious for a fully non-perturbative method that can determine the interaction between nucleons (or alternatively, the low energy constants of the effective theory) directly from QCD. At present, lattice QCD is the only such method.

Since nucleons are not infinitely heavy, the inter-nucleon potential is not a well defined quantity that can be measured on the lattice and then used in nuclear models. Instead, the connection between QCD and nuclear physics should be established through observables like scattering amplitudes and phase shifts, etc.. That brings out a problem: lattice calculations are done in euclidean space and analytic continuation of the euclidean correlation functions at infinite volume to Minkowski space is, in practice, impossible. This observation, known as Maiani-Testa theorem, seems to restrict lattice QCD to observables like masses, decays constants and amplitudes at kinematical thresholds. Phase shifts at some special values of the momenta can however be obtained by measuring the shifts in the low lying two-particle states due to the finite volume as long as the lattice size L is larger than the pion Compton wavelength (up to corrections of order ~ $e^{-m\pi L}$).

This can be intuitively understood by realizing that the baryon number two sector of QCD at momenta smaller than the pion mass reduces to a nonrelativistic quantum mechanical system with two nucleons interacting through contact interactions. At momenta Q much smaller than the ~ 1/a, where a is the nucleon-nucleon scattering length, this contact interaction is perturbative but it becomes strong at Q~ 1/a.

In particular, for lattices with size L much larger than the scattering length between the particles a the low lying states have typical momenta Q satisfying Q<< 1/a, and Luscher derived the formula relating the shifts in the energy levels and a as an expansion in powers of a/L. This method has been used to obtain pion-pion scattering phase shifts but in the two nucleon sector we are aware of only only one quenched calculation performed with a large pion mass.

In the two-nucleon case the condition L >> a can hardly be satisfied since the scattering lengths between two nucleons are large by QCD standards (5.42 fm in the spin triplet and 23.7 fm in the spin singlet channel) and numerical simulations with lattice sizes much larger than this are impractical. For L ~ a the shifts in the energy levels due to the nucleon-nucleon interactions are not small but can still be reliably computed and used to obtain information on the nucleon-nucleon interactions.

We have previously analyzed the two-nucleon case. There we found that, after taking into account the strong nucleon-nucleon interactions, lattice sizes L ~ 8 fm are necessary for the ground state to have small enough energy for the method to be valid, and even larger sizes if the excited states are considered. From the shift in the ground state energy the phase shifts at only one kinematical point can be determined. More handles on the phase shifts coming from the excited states would require even larger lattice sizes.

This paper proposes a method that i) allow smaller lattice sizes and ii) provides information about the phase shifts at any momenta. The basic idea is very simple: one just simulates the baryon number two sector of QCD in a finite torus and in the background of a fictitious "magnetic" potential with zero field strength, the kind of field generated by a thin solenoid going around inside the torus.

Due to the Aharonov-Bohm effect the energy levels are changed by this potential despite the fact that the field strength vanishes everywhere on the lattice. The strength of the potential can then be adjusted in order to have the ground state to have any energy desired. Alternatively we can describe the method as simulating QCD with twisted boundary conditions for the quark in one chosen spatial direction. The two descriptions are related by a change of variables amounting to a discontinuous gauge transformation.

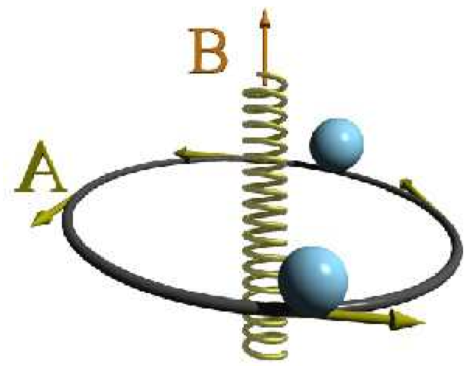

FIG. 1: Configuration of external field in the one-dimensional lattice case.

REFERENCES

[1] P. F. Bedaque, Phys. Lett. B593, 82 (2004).